INSTITUTE FOR RISK AND RELIABILITY

Leibniz Universität Hannover

Non-intrusive imprecise stochastic simulation (NISS) for uncertainty quantification

Introduction

Uncertainty quantification (UQ) has been widely accepted as an important task in many research fields. Commonly, two kinds of uncertainties, i.e., aleatory uncertainty and epistemic uncertainty, exist, where the former one is due to the intrinsic random property of parameters or events, and the later one is caused by the incompleteness of knowledge. Characterizing the above two kinds of uncertainties with a unified model framework, and propagating them through the computational models are two fundamental tasks of UQ.

Three categories of uncertainty characterization models, i.e., precise probability model (category I), non-probabilistic models (category II), and imprecise probability models (category III), have been developed for different situations (see figure 1), and the aim of this work is to develop a general methodology framework, named as non-intrusive imprecise stochastic simulation (NISS) for propagating these three categories of models, and specifically, for category II and III models, the interval model and parameterized p-box of the model are exemplified respectively. For simplicity, we take the estimation of failure probability function and bounds as examples. However, NISS can be used for estimating the bounds of any probabilistic responses (e.g., response distribution function and response moments).for more details, please refer to the references [1-3].

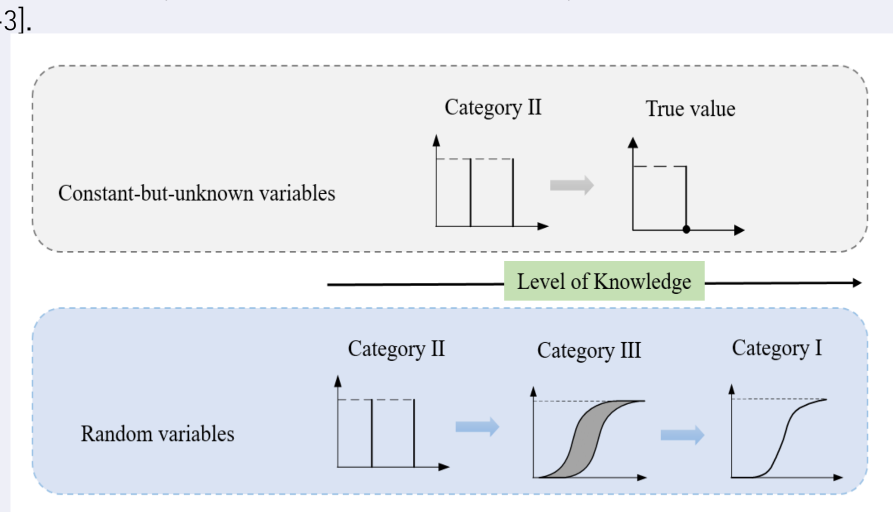

Figure 1: The roles of the three categories of uncertainty characterization models.

Non-intrusive Imprecise Stochastic Simulation

The rationale of NISS is shown in figure 2, where $\boldsymbol{y} = (y_1, y_2, \dots, y_m)^T$is the interval input vector (category II), $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_d)^T$ refers to the non-deterministic distribution parameters of category III input vector $\boldsymbol{x} = (x_1, x_2, \dots, x_n)^T$. The supports of $\boldsymbol{\theta}$ and $\boldsymbol{y}$ are assumed to be $[\underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}] = \times_{i=1}^{d} [\underline{\theta}_i, \overline{\theta}_i]$ and $[\underline{\boldsymbol{y}}, \overline{\boldsymbol{y}}] = \times_{i=1}^{m} [\underline{y}_i, \overline{y}_i]$ respctively. Then the failure probability P_f becomes a function of the epistemic parameters $\boldsymbol{\theta}$ and $\boldsymbol{y}$, whose supports determine the bounds of P_f. To implement the global NISS, independent auxiliary distributions should be assumed for each element of $\boldsymbol{\theta}$ and $\boldsymbol{y}$.

Then the failure probability function $\mathrm{P}_f(\boldsymbol{\theta}, \boldsymbol{y})$ can be decomposed by cut- or RS-HDMR as:

$$\begin{aligned}\mathrm{P}_f(\boldsymbol{\theta}, \boldsymbol{y}) = \ & \mathrm{P}_{f,0} + \sum_{1\le i\le d} \mathrm{P}_{f,\Theta_i}(\theta_i) + \sum_{1\le i<j\le d} \mathrm{P}_{f,\boldsymbol{\Theta}_{ij}}(\boldsymbol{\theta}_{ij}) \\ & + \sum_{1\le i\le m} \mathrm{P}_{f,\mathrm{Y}_i}(y_i) + \sum_{1\le i<j\le m} \mathrm{P}_{f,\mathbf{Y}_{ij}}(\boldsymbol{y}_{ij}) \\ & \sum_{1\le i\le d, 1\le j\le m} \mathrm{P}_{f,\Theta_i,\mathrm{Y}_j}(\theta_i, y_j) + \cdots + \mathrm{P}_{f,\boldsymbol{\Theta},\mathbf{Y}}(\boldsymbol{\theta}) \quad (1)\end{aligned}$$

where $\mathrm{P}_{f,0}$ is the constant component, $\mathrm{P}_{f,\Theta_i}(\theta_i)$ is the first-order component function of θ_i, etc.

As shown by figure 2, the second step (Simulation) of NISS is to estimate the constant component $\mathrm{P}_{f,0}$ by the well-established stochastic simulation techniques such as Monte Carlo simulation, subset simulation and line sampling. In this step, one can generate a group of samples of model inputs and outputs, based on which, the estimators of the component functions are derived in the third step (Estimation). The sensitivity indices corresponding to all component functions are also estimated without extra function call, and are used for identifying the influential components for synthesizing the mean estimate of $\mathrm{P}_f(\boldsymbol{\theta}, \boldsymbol{y})$, as well as for measuring the error due to HDMR truncation.

Based on the mean estimate of $\mathrm{P}_f(\boldsymbol{\theta}, \boldsymbol{y})$, the bounds of P_f can be easily estimated by any optimization solver. Generally, in high-dimensional applications, most epistemic parameters are not influential, thus their component functions can be neglected. In many cases, $\mathrm{P}_f(\boldsymbol{\theta}, \boldsymbol{y})$ is additive to some of the influential epistemic parameters, making it possible to simplify the optimization problem into low-dimensional (even univariate) problems. The above features make NISS effective for propagating the three categories of models especially when the input dimension is high.

If cut-HDMR is utilized, it is called local NISS; otherwise, it is termed as global NISS. The key of NISS is to derive the estimators for component functions based on the samples of stochastic simulation for estimating $\mathrm{P}_{f,0}$. The formulations of these estimators differ when different stochastic simulation techniques and/or different HDRM decompositions are used, but their statistical properties remain the same as for the estimator of $\mathrm{P}_{f,0}$. This feature makes it quite easy to estimate the statistical errors of the estimates of $\mathrm{P}_f(\boldsymbol{\theta}, \boldsymbol{y})$ as well as the resultant bounds.

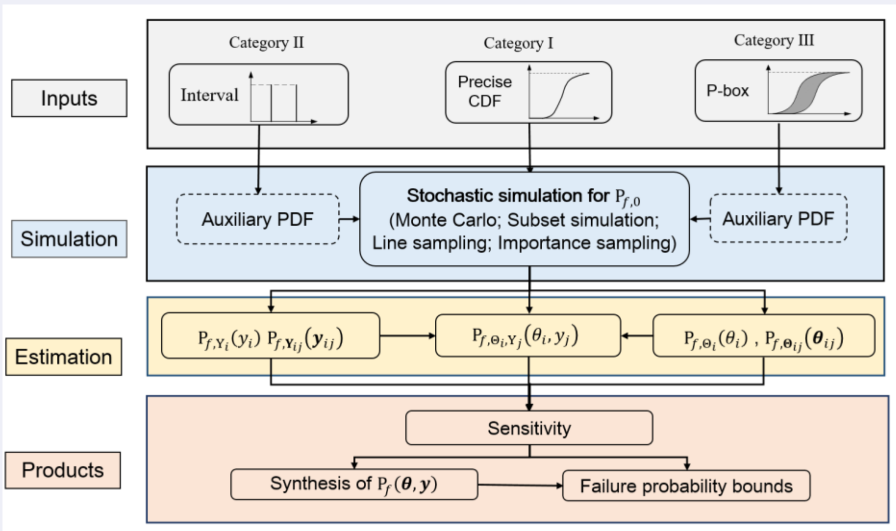

Figure 2: Rationale of NISS

Reference

[1] Wei, P.F. et al. (2019). Non-intrusive stochastic analysis with parameterized imprecise probability models: I. Performance estimation. MSSP, 124,349-368.

[2] Wei, P.F. et al. (2019). Non-intrusive stochastic analysis with parameterized imprecise probability models: II. Reliability and rare events analysis. MSSP 126. 227-247.

[3] Song, J.W. et al. (2019). Generalization of non-intrusive imprecise stochastic simulation for mixed uncertain variables. MSSP

Note

This work is supported by the Alexander von Humboldt Foundation.

Alexander von Humboldt Stiftung/Foundation

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