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# DECISION-MAKING FOR RESILIENCE-ENHANCING ENDOWMENTS IN COMPLEX SYSTEMS USING PRINCIPLES OF RISK MEASURES

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Complex systems, such as turbines, industrial plants and infrastructure networks are of paramount importance to modern communities. However, these systems are subject to a plethora of different threats. Therefore, novel developments are focused not only on increasing the robustness and reliability of systems but also on taking into account their recovery. The concept of resilience encompasses these developments. An essential aspect concerning the quantification of resilience is how it can help decision-makers to efficiently improve and construct the complex systems of our modern communities. Consequently, it is necessary to develop comprehensive and widely adaptable, resilience-based decision-making tools. In this paper, a numerically efficient method aiding decision-makers in balancing between different resilience-enhancing investments is presented. Using an appropriate resilience metric, and moreover an adapted systemic risk measure, the approach allows direct comparison between failure prevention arrangements and recovery improvement arrangements, leading to an optimal tradeoff relative to the resilience of a system. Additionally, the method is capable of incorporating monetary aspects into the decision-making process.

*Keywords*: Resilience, decision-making, systemic risk measure, tradeoff.

## 1 Introduction

Modern societies depend on a variety of complex systems, such as turbines, industrial plants or infrastructure networks. These form complex capital goods whose construction, improvement and regeneration are of paramount importance for society. However, these systems are subject to a plethora of different threats. Evidence shows that a wide range of natural, technical and anthropogenic impacts at all scales can severely affect the functionality of these systems. Due to the quickly growing complexity, it is extremely difficult to identify all possible criterial impacts and to prevent them accordingly. Therefore, novel developments are required to focus not exclusively on increasing the robustness and reliability of systems, but also on taking into account their recovery (Tran et al. 2017, Linkov & Palma-Oliveira 2017). The concept of resilience encompasses these developments by considering and optimizing robustness, reliability and recovery of systems not only from technical, but also from economic perspectives (Cimellaro et al. 2010, Ayyub 2015). This leads to a paradigm shift from a strategy that secures systems from failing to a strategy that makes systems effective also in the case of failure. During the last two decades, a vast number of different approaches to quantify resilience were published (Bergström et al. 2015, Hosseini et al. 2016).

An essential aspect introduced by this new approach is how the quantification of resilience can help decision-makers to efficiently design and improve the key complex systems present all over our modern communities (Hosseini et al. 2016, Tran et al. 2017). It is obvious that without the consideration of monetary constraints, the resilience of a system can be vastly increased. However, neglecting monetary constraints does not reflect reality, and it is necessary to develop methods that help decision-makers balance between different resilience-enhancing investments. Therefore, in this work, the sophisticated resilience metric by Ouyang et al (2012) as well as an adapted systemic risk measure by Feinstein et al. (2016) are applied to an exemplary flow system to present an efficient method for determining the most cost-effective combination of different resilience-enhancing investments under a certain minimum-resilience condition.

In the following, section 2 briefly describes the resilience metric by Ouyang et al (2012) and section 3 introduces the adaptation of the systemic risk measure by Feinstein et al. (2016). Section 4 provides an overview of the simulation procedures and the exemplary flow network. The paper finishes with a discussion about the results in section 5 and conclusions in the final section.

## 2 Resilience Metric

The expected annual resilience metric $Re$ by Ouyang et al. (2012) is defined as the expectation of the ratio between the integral of the system performance $P(t)$ over a target time interval $[0, T]$ and the integral of the target system performance $TP(t)$ in the same interval. According to Hosseini et al. (2016), it is categorized as a probabilistic and time-dependent resilience metric and is defined as:

$$Re = E[Y], \qquad \text{where} \qquad Y = \frac{\int_0^T P(t)dt}{\int_0^T TP(t)dt}. \tag{1}$$

$P(t)$ is a random quantity modeled by a stochastic process. $TP(t)$ is generally considered as a stochastic process as well, but it is assumed to be a non-random constant $TP$ in this work. By introducing the discrete number of failure events in the target time $N(T)$, Eq. (1) could be further written as

$$Y = 1 - \frac{\sum_{n=1}^{N(T)} AIA_n(t_n)}{TP \cdot T}, \tag{2}$$

where $t_n$ is the random time of the occurrence of the $n$th event. Finally, $AIA_n(t_n)$ is the impact area, i.e., the area between the reduced system performance curve caused by the $n$th failure event and the target system performance curve.

Although this metric is capable of considering hazards of different types, for illustrative purposes, only one hazard type is taken into account. The resilience metric takes values between 0 and 1, where $Re = 1$ indicates a system performance corresponding to the target performance over the regarded time period $[0, T]$, and $Re = 0$ indicates that the system is not working over $[0, T]$.

## 3 Systemic Risk Measure

The adaptation of the approach suggested by Feinstein et al. (2016) quantifies system inherent risk on the basis of descriptive input-output models and acceptance criterions representing the arbitration of a regulatory authority.

Let $(\Omega, \mathcal{F}, P)$ be a probability space, $l \in \mathbb{N}$ the entities number of a considered system and $k_i^T \in \mathbb{R}^n$ a vector of control variables specifying to the $i$th entity. For any scenario $\omega \in \Omega$ and endowment $k$ assume $Y_k(\omega)$ to be the real number capturing the ratio in Eq. (2) applied to the investigated system which is assumed to be increasing in $k$. In this case the underlying input-output model is provided by $Y = (Y_k)_{k \in \mathbb{R}^{l \times n}}$.

According to Feinstein et al. (2016), acceptance criterions $\mathcal{A} \subseteq \mathcal{X}$ are sets of random variables meeting the requirements of a decision-maker, with $\mathcal{X}$ denoting a suitable vector space of random variables, e.g. the family of all bounded random variables noted $L^\infty(\Omega, \mathbb{R})$. In the context of this work, for an acceptance threshold $\lambda \in (0,1)$, the acceptance criterion is defined as:

$$\mathcal{A} = \{X \in \mathcal{X} | E[X] \geq \lambda\}. \tag{1}$$

Once the input-output model and the acceptance criterion are determined the adapted, set-valued and multivariate risk measure is defined by:

$$R(Y; k) = \{m \in \mathbb{R}^{l \times n} | Y_{k+m} \in \mathcal{A}\}. \tag{2}$$

These sets consist of all endowment-enhancing $m$ added to $k$, with $k$ being understood as basic equipment of the regarded system, leading to a resilience value greater or equal to $\lambda$, see Eq. (2). Hereinafter the basic equipment $k$ is set to zero and $R(Y; 0)$ is written as $R(Y)$. Consequently, all elements $a$ of this set are denoted as endowments.

In Feinstein et al. (2016) the authors present a grid search algorithm to efficiently approximate $R(Y; k)$ with a chosen accuracy, cf. Audet & Hare (2017).

## 4 Flow Network and Procedures to Simulate the System Performance

Flow networks are generically applicable models used to represent complex systems, such as turbines, industrial plants or infrastructure networks. In this application the flow network shown in Figure 1, with seven nodes and eight edges, is considered. In this case, the edges are considered to be the essential network components, each of which is associated with one out of $b$ component types. Thereby the endowment of each edge $i \in \{1, \ldots, 8\}$ is designated by $a_i^j = (c_j, r_j^*) \in \mathbb{N}^{1 \times n}$ with $j \in \{1, \ldots, b\}$, containing $n = 2$ properties, capacity $c_j$ and recovery improvement $r_j^*$. In addition to the edge grouping, further restrictions can be made, e.g. by setting the component property $r^*$ to be constant. Analogous to Feinstein et al. (2016), these constraints can be captured by narrowed elements $a' \in \mathbb{N}^p$ and a monotonously increasing function $g: \mathbb{N}^p \to \mathbb{N}^{l \times n}$, with $p$ as the number of non-restricted entities of the native endowment matrix $a$. The systemic risk measure results as: $R(Y) = \{m \in \mathbb{N}^p | Y_{g(m)} \in \mathcal{A}\}$. Note that under these assumptions $R(Y)$ is a discrete set and the abovementioned grid search algorithm by Feinstein et al. (2016) no longer provides an approximation but determines $R(Y)$ exactly. The source node of the flow network is denoted by $s$ with an initial flow $w$ and the target node by $t$ with a destination flow $v$, respectively.

The considered time interval $[0, T]$ is subdivided into $u$ time steps $t_0, \dots, t_T$. In each time step $t_h$ the system performance is determined by the ratio of the time-dependent destination flow $v$ to the initial flow $w$, i.e. $P(t) = v(t_h)/w$ with $t \in [t_h, t_{h+1})$.

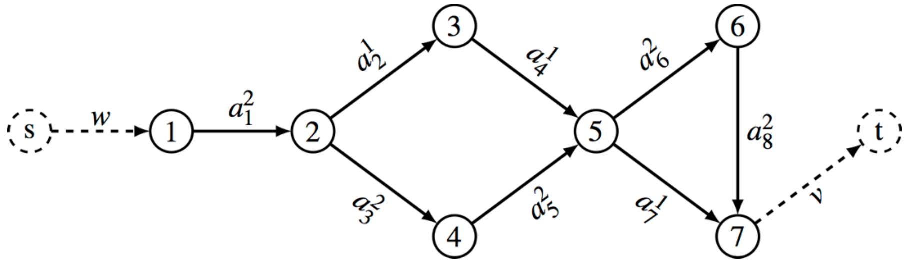


Figure 1. Exemplary flow network for $b = 2$ component types.

The flow for a given endowment $a$ is simulated as follows: In each time step, the allocation of the initial flow is determined node by node, based on a breadth-first search. The procedure starts with the source node and takes into account the following rules: (i) the incoming flow is allocated to all subsequent edges in proportion to the respective capacities; (ii) if the capacity of a subsequent edge is exceeded, this edge is considered to be destroyed immediately and the flow is instead allocated to the remaining edges, taking into account (i); (iii) if a node has no subsequent edge, its current flow is lost.

The failure probability of each edge depends on the respective utilization of the edge capacity caused by the flow as $p_i = \beta \cdot (v_i/c_i)$, where $v_i$ is the current flow and $c_i$ is the capacity of the respective edge $i$. The factor $\beta \in (0,1)$ mitigates the failure probability. At the end of the time step, the failure probability of each edge is determined and according to this, failures are realized. A failed node is considered to be destroyed. Note that the failure process in this work is assumed to be immediate and other failure profiles or aging effects, as e.g. introduced by Ayyub (2014), are not taken into account for reasons of simplification. The same applies to the recovery profiles. Each destroyed edge is assumed to be immediately recovered after $r = r_{\max} - r^*$ time steps, with $r^* < r_{\max}$, where $r_{\max}$ is the maximum number of time steps for recovery. This corresponds to the one step recovery profile, introduced by Ayyub (2015).

The described simulation procedure is performed $u$ times, once for each time step, resulting in a staircase-shaped system performance over $[0, T]$. Furthermore, this procedure is repeated 1000 times so that the arithmetic mean of the ratio between the system performance $P(t)$ and the target performance $TP(t)$ shown in Eq. (1) converges to the expected value, that is the resilience. This scheme is repeated according to the grid search algorithm mentioned in section 3, providing the set of all accepted endowments $R(Y)$. In the subsequent section, two different system scenarios will be presented and discussed.

## 5 Scenarios and Decision-Making

The method described in the previous sections, is applied to two scenarios of the system shown in Figure 1. The values of the decisive parameters for both scenarios are assumed to be: acceptance threshold $\lambda = 0.8$, mitigation factor $\beta = 0.025$, number of time steps $u = 100$. Note that all parameter values in this section are chosen arbitrarily for illustrative purposes and that the costs for the endowments properties, in both scenarios, are assumed to be linearly increasing.

### 5.1 *Scenario 1*

In the first scenario, the recovery improvements of both component types are assumed to be constant with $r_1^* = r_2^* = 11$. This leads, with an assumed maximum recovery time $r_{\max} = 21$, to a constant recovery duration for each destroyed edge of $r = 10$ time steps. The capacities of both edge types are explored over $c_1, c_2 \in \{1, \dots, 20\}$. From this, and with an initial flow $w = 3$ the set of all accepted endowments $R(Y)$ results as shown in Figure 2a), where the filled dots represent the elements of $R(Y)$.

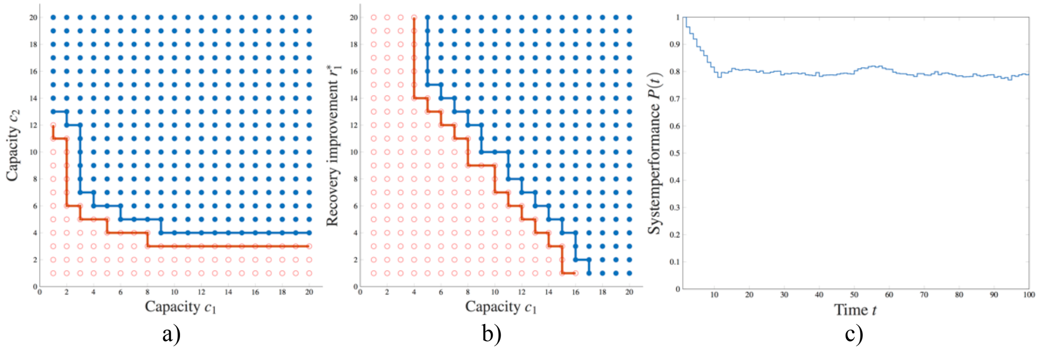


Figure 2. a) Accepted endowments of scenario 1 (filled dots); b) accepted endowments of scenario 2 (filled dots); c) staircase-shaped system performance $P(t)$ over $[0, T]$ of scenario 2 for endowment $a^1 = (9{,}10)$.

### 5.2 *Scenario 2*

In the second scenario, all eight edges are assumed to be assigned to one component type. As in scenario one, the maximum recovery time is set to $r_{\max} = 21$, and the recovery-improvement is explored over $r_1^* \in \{1, \dots, 20\}$. In addition, the capacity is explored over $c_1 \in \{1, \dots, 20\}$, as well. This leads, with an initial flow $w = 5$, to a set of all accepted endowments $R(Y)$ as shown in Figure 2b). Figure 2c) shows the staircase-shaped system performance $P(t)$ over the time interval $[0, T]$ for an exemplary endowment $a^1 = (c_1, r_1^*) = (9{,}10)$.

### 5.3 *Decision-Making*

In resilience-enhancing decision-making, the monetary conditions must be considered. Therefore, it is necessary to find the most cost-effective endowment $\hat{a}$. Under the assumption of linear increasing endowment properties costs, $\hat{a}$ is an element of $\bar{R}(Y) \subseteq R(Y)$, with $\bar{R}(Y)$ representing the set of all endowments located at the upper frontier graph as identified in Figure 2a) and 2b). Note that due to the monetary linearity condition, this endowment in $\bar{R}(Y)$ can only correspond to a non-dominated point in the upper front graph, cf. efficient allocation rules (EAR) by Feinstein et al. (2016). The consideration of these conclusions and the application of the grid search algorithm lead to a high efficiency in terms of the computational effort.

Considering endowment properties costs of $c_{1,cost} = 250€$, $c_{2,cost} = 180€$, $r_{1,cost}^* = r_{2,cost}^* = 170€$, the most cost-effective endowment for the first scenario results as $\hat{a}^1 = (c_1, r_1^*) = (4{,}11)$, $\hat{a}^2 = (c_2, r_2^*) = (6{,}11)$, with a total cost of $23360€$. For the second scenario, with endowment properties costs of $c_{1,cost} = 200€$, $r_{1,cost}^* = 170€$ the most cost-effective endowment results as $\hat{a}^1 = (c_1, r_1^*) = (9{,}10)$, and a total cost of $28000€$.

## 6 Conclusion

In this paper, a decision-making procedure was introduced that allows to identify optimal tradeoffs between resilience-enhancing endowments in complex systems. It is based on an appropriate systemic risk measure. Using a suitable resilience metric, the paper demonstrates that this approach enables a direct comparison of the impact of different controls on the resilience of the system, e.g. failure prevention and recovery improvement arrangements. In addition, the method is capable of incorporating monetary aspects into this decision-making process – which are in reality of paramount importance. Furthermore, the computational effort is significantly reduced by a grid search algorithm for systemic risk measures. Another benefit of the suggested methodology is its broad applicability that is not limited to flow networks. The approach can easily be adapted to other systems, e.g. systems whose performance purely depends on the topology. The presented method is capable to substantially support decision-makers in improving the complex systems of our modern society and increasing their resilience.

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