Efficient Reliability Analysis of an Axial Compressor in Consideration of Epistemic Uncertainty

Julian Salomon[1]
E-mail: *salomon@irz.uni-hannover.de*

Niklas Winnewisser[1]
E-mail: *winnewisser@irz.uni-hannover.de*

Pengfei Wei[1,2]
E-mail: *pengfeiwei@nwpu.edu.cn*

Matteo Broggi[1]
E-mail: *broggi@irz.uni-hannover.de*

Michael Beer[1,3,4]
E-mail: *beer@irz.uni-hannover.de*

[1]*Institute for Risk and Reliability, Leibniz University Hanover, Germany.*
[2]*School of Mechanics, Civil Engineering and Architecture, Northwestern Polytechnical University, Xi'an, CN.*
[3]*Institute for Risk and Uncertainty, University of Liverpool, Peach Street, Liverpool, UK.*
[4]*International Joint Research Center for Engineering Reliability and Stochastic Mechanics (ERSM), Tongji University, Shanghai, CN.*

The reliability of complex systems, e.g. infrastructure networks or turbines, and further its proper prediction, is of paramount importance to our modern society. The concept of survival signature enables to entirely separate the structure of a network and its components probabilistic properties. As a consequence, once survival signature of a network has been computed, a reliability analysis can be performed by evaluating only the probabilistic part of the network. It is precisely this advantage that makes the analysis particularly efficient and clearly distinguishes the concept from traditional approaches.
In reality, epistemic uncertainties, as vague or varying expert knowledge on a component's future behavior, impede a proper reliability analysis. Considering and quantifying these in a plausible manner is therefore of major interest to decision-makers. Fuzzy probabilities offer a naturally fitting approach for this purpose. Despite the advantageous efficiency of the survival signature, analyzing complex systems under considertaion of epistemic uncertainties implies high computational effort. In order to counteract these efforts, two extended Monte Carlo simulation methods, originally utilized for imprecise structural reliability problems, are adapted into the context of system reliability.
By integrating fuzzy probabilities into the probability structure of the survival signature and employing advanced Monte Carlo methods, this paper provides an efficient approach to quantify the reliability of a complex system taking into account epistemic uncertainties. Beyond the merging of the theoretical aspects, the approach is applied to a functional model of an axial compressor, in order to demonstrate its usability.

Keywords: survival signature, system reliability, network reliability, reliability analysis, epistemic uncertainty, fuzzy probabilities, extended Monte Carlo methods, Non-intrusive Imprecise Stochastic Simulation

1. Introduction

Engineering systems are pervasive elements of modern societies. Examples are infrastructure networks, industrial plants or machines, e.g., gas turbines. For economic and safety-related reasons, sufficient reliability of such complex capital goods has to be guaranteed. In practice, those systems deteriorate due to environmental and operational influences. As a consequence, the overall system performance decreases. To ensure reliability, i.e.,

Proceedings of the 30th European Safety and Reliability Conference and the 15th Probabilistic Safety Assessment and Management Conference
Edited by Piero Baraldi, Francesco Di Maio and Enrico Zio

ISBN/DOI: 978-981-14-8593-0

continuing functionality, respectively, in case of critical deterioration, its prompt recovery, the system's design, operation and maintenance must be chosen accordingly. Therefore, the time-variant quantification of a system's reliability is of utmost importance.

An efficient approach to modeling the reliability of systems with multiple component types is the concept of survival signature, introduced and discussed in Coolen and Coolen-Maturi (2013, 2016). Its key advantage over traditional approaches is provided by a clear separation of the system's structure and the probabilistic properties of its components. Thereby, this approach reduces the computational effort caused by repeated evaluations that are typically required in design and maintenance processes, Patelli et al. (2017).

In reality, the characterization of component behavior is intrinsically conditioned by epistemic uncertainties, e.g., due to the analysts' subjective perspective, Der Kiureghian and Ditlevsen (2009). For practical reasons, these reducible uncertainties, often referred to as imprecision, can be implemented in the model in set-theoretical form, Aughenbaugh and Paredis (2005). For decision-makers it is of utmost importance to identify a critical level of these imprecisions. As proposed in Beer et al. (2013), this can be realized by utilizing fuzzy probabilities.

The consideration of epistemic uncertainties, however, yields in an enormous increase of computational effort, Hofer et al. (2002). In Wei et al. (2019a), the Local Extended Monte Carlo Simulation (LEMCS) and the Global Extended Monte Carlo Simulation (GEMCS), summarized as Non-Intrusive Stochastic Simulation (NISS), are introduced to efficiently compute imprecise structural models. In the present work, these basic NISS methods are adapted to the context of system reliability by means of the concept of survival signature, while fuzzy probabilities are applied to model imprecision in the probability structure. Thereby, this paper provides a novel approach for an efficient computation of a system's reliability including imprecision.

The paper proceeds as follows: Section 2 briefly reviews the fundamental theory of survival signature, fuzzy probability and NISS method. Based on this, Section 3 develops the proposed novel approach. In Section 4 the method is applied to a functional model of a multi-stage high-speed axial compressor. Section 5 summarizes the results and discusses questions of future research.

2. Theoretical Fundamentals

2.1. *Survival Signature*

The survival signature according to Coolen and Coolen-Maturi (2013) is a concept for efficiently determining the time-dependent reliability of systems, that are composed of components of different types. More detailed information about the concept and its derivation can be found, e.g. in Coolen and Coolen-Maturi (2013, 2016); Feng et al. (2016).

2.1.1. *Structure Function*

Suppose a system composed of m components of a single type. Then, $\underline{x} = (x_1, x_2, \ldots, x_m) \in \{0,1\}^m$ defines the state vector of these components with $x_i = 1$ indicating a functioning state of the ith component and $x_i = 0$ indicating a non-functioning state. The structure function ϕ is a function of the state vector, describing the working state of the regarded system: $\phi = \phi(\underline{x}) : \{0,1\}^m \to \{0,1\}$. Accordingly, $\phi(\underline{x}) = 1$ indicates a functioning system and $\phi(\underline{x}) = 0$ indicates a non-functioning system.

Now suppose a system composed of components of multiple types $K \geq 2$, then the number of system components is given by $m = \sum_{k=1}^{K} m_k$ with m_k denoting the number of components of type $k \in \{1, 2, \ldots, K\}$. Assuming the failure times of components of the same type to be independent and identically distributed (iid), the state vector for multiple types can be defined, equivalent to systems with only a single component type, as $\underline{x}^k = \left(x_1^k, x_2^k, \ldots, x_{m_k}^k\right)$.

2.1.2. *Survival Signature and Survival Function*

The survival signature describes the probability of a system being in a functioning state, purely depending on the number of functioning components l_k for each type k. Assuming the failure times of components of the same type to be (iid) and exchangeable within this type, the survival signature can be defined as:

$$\Phi\left(l_1, l_2, \ldots, l_K\right) = \left[\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1}\right] \times \sum_{\underline{x} \in S_{l_1, l_2, \ldots, l_K}} \phi(\underline{x}), \tag{1}$$

with $\binom{m_k}{l_k}$ denoting the total number of state vectors $\underline{x}^k$ of type k and $S_{l_1, l_2, \ldots, l_K}$ denoting the set of all state vectors of the entire system for which $l_k = \sum_{i=1}^{m_k} x_i^k$. Note that the survival signature only depends on the topology of the system, regardless of any time-dependent failure behavior of its components.

Let $C_k(t) \in \{0, 1, \ldots, m_k\}$ denote the number of components of type k in a working state at time t and suppose the probability distribution for the failure times of type k to be known with $F_k(t)$, being the corresponding cumulative distribution function. Then,

$$\begin{aligned} &P\left(\cap_{k=1}^{K}\left\{C_k(t)=l_k\right\}\right) \\ &=\prod_{k=1}^{K} P\left(C_k(t)=l_k\right) \\ &=\prod_{k=1}^{K}\binom{m_k}{l_k}\left[F_k(t)\right]^{m_k-l_k}\left[1-F_k(t)\right]^{l_k} \end{aligned} \tag{2}$$

describes the system's probabilistic structure, i.e. the time-dependent failure behavior of the system components, regardless of the system's topology. The survival function, describing the probability of a regarded system being in a functioning state at time t, results as:

$$\begin{aligned} P\left(T_s>t\right)=&\sum_{l_1=0}^{m_1}\ldots\sum_{l_k=0}^{m_k}\Phi\left(l_1,l_2,\ldots,l_K\right) \\ &\times P\left(\cap_{k=1}^{K}\left\{C_k(t)=l_k\right\}\right), \end{aligned} \tag{3}$$

with T_s denoting the random system failure time. Note that it is precisely the fact of separating the topology and the time-dependent probability structure of a system that makes the survival signature so unique and advantageous.

2.2. *Fuzzy Probability*

In system reliability analysis for engineering problems, epistemic uncertainties frequently occur, e.g. in form of expert knowledge regarding the underlying distribution functions of the failure probabilities of system components. Fuzzy probability theory enables to take these uncertainties into account.

Let $F(x)$ be a probability distribution function, describing the failure probability of a system component up to time x. Further, assume that the knowledge of the parameters of this distribution function is not precise, but contains epistemic uncertainty. Then Fig. 1 shows the fuzzy probability distribution function $\tilde{F}(x)$, describing this phenomenon, with $\mu(F(x))$ denoting the membership function of $F(X)$ and $supp(\tilde{F}(x)) = [F_{\alpha=0,low}(x), F_{\alpha=0,up}(x)]$ denoting the support of $\tilde{F}(x)$. Note that for $\mu(F(x)) = 1$, corresponding to a α-level of $\alpha = 1$, $\tilde{F}(x) = F(x)$. Comprehensive information on fuzzy probability and its practical applications is provided, e.g. in Möller et al. (2001); Buckley (2005); He et al. (2015).

2.3. *Non-Intrusive Imprecise Stochastic Simulation*

The Non-Intrusive Imprecise Stochastic Simulation (NISS), according to Wei et al. (2019a) and Wei et al. (2019b), provides a general methodological framework for propagating parameterized imprecise probability models through a black-box simulator with only one stochastic simulation, and theoretically, any stochastic simulation algorithm can be injected into this framework to tackle different types of problems. In this work, the two basic NISS methods, LEMCS and GEMCS, are reviewed, where LEMCS is the basis of all local NISS methods, while GEMCS provides basis for all global NISS methods. The NISS methods are originally developed for performance and reliability estimation of structures simulated with a black-box model, e.g. a finite element model. In this work, both methods are adapted for an utilization in the context of system reliability by means of the concept of survival signature.

3. Proposed Methology

Let $\boldsymbol{t} = (t_1, t_2, \ldots, t_n)^\top$ denote the failure times of a system's components, and t_s indicates the failure time of the system. For a coherent system, a non-decreasing deterministic g-function, denoted as $t_s = g(\boldsymbol{t})$, can be uniquely derived for modeling the relationship between system and component failure times. The failure times of all component functions are intrinsically random variables, and the joint density function is assumed to be $f(\boldsymbol{t}\,|\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ indicates the m-dimensional vector of non-deterministic distribution parameters modeled by independent fuzzy sets with support $[\boldsymbol{\theta}_l, \boldsymbol{\theta}_u]$. The epistemic uncertainty embodied through $\boldsymbol{\theta}$ might result from a lack of life data on components or expert knowledge, and supports can be inferred by, e.g., confidence interval estimation. Based on the above setting, the system failure time is also a random variable with non-deterministic distribution parameters, where the probability distribution reflects the natural variation of system failure time, and the bounds of probability reflect the degree of unknown on this variation. The system survival function can then be formulated as:

$$R_s\left(t_s|\boldsymbol{\theta}\right)=\int_{\mathbb{R}+} I\left[g\left(\boldsymbol{t}\right)>t_s\right] f\left(\boldsymbol{t}|\boldsymbol{\theta}\right)\mathrm{d}\boldsymbol{t}, \tag{4}$$

where $\mathbb{R}^+$ indicates the space of non-negative real numbers, and $I[\cdot]$ is the indicator function with the values being either one if the argument is true, or zero if it is false. With the above setting, the system survival function can be reformulated as:

$$\begin{aligned} &R_s\left(t_s|\boldsymbol{\theta}\right)= \\ &\int_{\mathbb{R}+} I\left[g\left(\boldsymbol{t}\right)>t_s\right]\frac{f\left(\boldsymbol{t}|\boldsymbol{\theta}\right)}{f\left(\boldsymbol{t}|\boldsymbol{\theta}^*\right)} f\left(\boldsymbol{t}|\boldsymbol{\theta}^*\right)\mathrm{d}\boldsymbol{t}, \end{aligned} \tag{5}$$

where $\boldsymbol{\theta}^*$ can be any fixed and crisp point of $\boldsymbol{\theta}$. Then, given a set of random samples $\boldsymbol{t}^{(k)}$

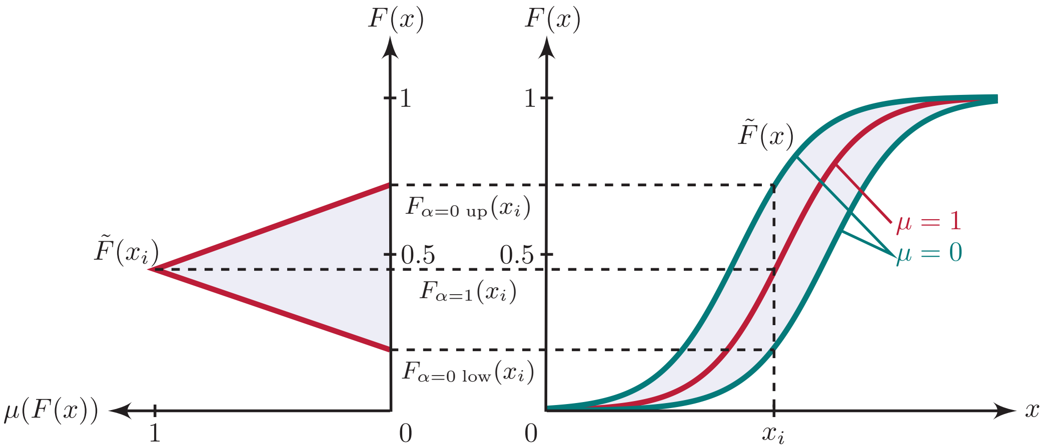

Fig. 1. Fuzzy probability distribution function of a continuous fuzzy random variable; adapted from Ref. Möller et al. (2001)

$(k = 1, 2, \ldots, N)$ following $f(\boldsymbol{t}|\boldsymbol{\theta}^*)$, the LEMCS estimator of the system survival function is given as:

$$\hat{R}_s(t_s|\boldsymbol{\theta}) = \frac{1}{N}\sum_{k=1}^{N} I\left[g\left(\boldsymbol{t}^{(k)}\right) > t_s\right] \frac{f\left(\boldsymbol{t}^{(k)}|\boldsymbol{\theta}\right)}{f\left(\boldsymbol{t}^{(k)}|\boldsymbol{\theta}^*\right)}. \tag{6}$$

This estimator is unbiased, and its variance can be easily derived. With the above estimator, the bounds of the survival function can be computed by any global optimization algorithm, such as genetic algorithms and particle swarm algorithms.

The GEMCS method involves first attributing auxiliary distributions for $\boldsymbol{\theta}$, which, in the simplest case, can be uniform distributions within $[\boldsymbol{\theta}_l, \boldsymbol{\theta}_u]$. Let $p(\boldsymbol{\theta})$ denote the joint density function of these auxiliary distributions. Then a set of joint random samples $\left(\boldsymbol{t}^{(k)}, \boldsymbol{\theta}^{(k)}\right)$ can be generated following the joint density $f(\boldsymbol{t}|\boldsymbol{\theta})\, p(\boldsymbol{\theta})$, based on which the GEMCS estimator for the system survival function results as:

$$\hat{R}_s(t_s|\boldsymbol{\theta}) = \frac{1}{N}\sum_{k=1}^{N} I\left[g\left(\boldsymbol{t}^{(k)}\right) > t_s\right] \frac{f\left(\boldsymbol{t}^{(k)}|\boldsymbol{\theta}\right)}{f\left(\boldsymbol{t}^{(k)}|\boldsymbol{\theta}^{(k)}\right)}. \tag{7}$$

Both, the LEMCS and GEMCS performance, might vary for different types of probability distributions or different distribution parameters and depend on an appropriate choice for $\boldsymbol{\theta}^*$ and $p(\boldsymbol{\theta})$, respectively. More detailed information can be obtained in Wei et al. (2019a).

Another key feature of the classical NISS method is the high-dimensional model representation (HDMR), based on which the behavior of the system survival function with respect to $\boldsymbol{\theta}$ can be learned visibly, and the variation of estimators can be substantially reduced, especially when the number of components with imprecise distribution parameters is large. However, in this paper, the LEMCS and GEMCS estimator are solely utilized without HDMR decomposition.

3.1. *LEMCS Algorithm*

A modified version of the MCS algorithm 2 in Patelli et al. (2017) is utilized as the stochastic simulation module for implementing LEMCS and GEMCS. The LEMCS algorithm is then described as follows:

Step A1: Discretize the support $[0, \bar{t}]$ of system failure time uniformly as $0 = t_{s1} < t_{s2} < \ldots < t_{sd} = \bar{t}$, and initialize the value of $\boldsymbol{\theta}^*$ and the number N of deterministic simulations. Let $k = 1$.

Step A2: Sample the failure time $\boldsymbol{t}^{(k)} = \left(t_1^{(k)}, t_2^{(k)}, \ldots, t_n^{(k)}\right)$ for all components following $f(\boldsymbol{t}|\boldsymbol{\theta}^*)$ randomly.

Step A3: At each time instant t_{si}, count the number of components working for each component type as $C_j(t_{si})$, where $j = 1, 2, \ldots, K$ denotes the component type.

Step A4: Evaluate the survival signature at each time instant as $\Phi_{si}^{(k)} = \Phi\left(C_1(t_{si}), C_2(t_{si}), \ldots, C_K(t_{si})\right)$.

Step A5: Define the weight function for the sample $\boldsymbol{t}^{(k)}$ as $w^{(k)}(\boldsymbol{\theta}) = \frac{f\left(\boldsymbol{t}^{(k)}|\boldsymbol{\theta}\right)}{f\left(\boldsymbol{t}^{(k)}|\boldsymbol{\theta}^*\right)}$. If $k = N$, finish the simulation; else, let $k = k + 1$, and go back to Step A2.

Based on the samples $\Phi_{si}^{(k)}$, the LEMCS estimator for the system survival function at time t_{si} is formulated as:

$$\hat{R}_s\left(t_{si}|\boldsymbol{\theta}\right) = \frac{1}{N}\sum_{k=1}^{N}\Phi_{si}^{(k)}w^{(k)}\left(\boldsymbol{\theta}\right). \tag{8}$$

Computing at each time instant the minimum and maximum values of the estimator in Eq. 8, by utilizing any global optimization algorithm, leads to the estimated lower and upper bound of the system survival function.

3.2. *GEMCS Algorithm*

The GEMCS algorithm is similar to the LEMCS algorithm except that the stochastic simulation needs to be implemented in the joint space of $\boldsymbol{t}$ and $\boldsymbol{\theta}$. Given the auxiliary density function $p\left(\boldsymbol{\theta}\right)$, the GEMCS algorithm is described as follows:

Step B1: Discretize the support $[0, \bar{t}]$ of system failure time uniformly as $0 = t_{s1} < t_{s2} < \ldots < t_{sd} = \bar{t}$, and initialize the number N of deterministic simulation. Let $k = 1$.
Step B2: Generate a joint random sample $\left(\boldsymbol{t}^{(k)}, \boldsymbol{\theta}^{(k)}\right)$ following the joint density $f\left(\boldsymbol{t}|\boldsymbol{\theta}\right)p\left(\boldsymbol{\theta}\right)$.
Step B3: Same as Steps A3 and A4.
Step B4: Evaluate the weight function for the joint sample $\left(\boldsymbol{t}^{(k)}, \boldsymbol{\theta}^{(k)}\right)$ as $w^{(k)}\left(\boldsymbol{\theta}\right) = \frac{f\left(\boldsymbol{t}^{(k)}|\boldsymbol{\theta}\right)}{f\left(\boldsymbol{t}^{(k)}|\boldsymbol{\theta}^{(k)}\right)}$. If $k = N$, finish the simulation; else, let $k = k + 1$, and go back to Step B2.

The GEMCS estimator for the system survival function is formulated equivalently to the LEMCS estimator in Eq. 8, and the estimated lower and upper bound of the system survival function can be computed at each time instant by utilizing any optimization algorithm.

The most appealing aspect of both, the LEMCS and GEMCS algorithm, is that only a single stochastic simulation is required in order to deal with the epistemic uncertainties. Therefore, the typically necessary double loop simulation can be avoided. Besides the advantageous properties of the survival signature, it is precisely this feature, that makes the proposed method so efficient and clearly distinguishes it from traditional approaches.

3.3. *Repeated P-box Analysis for Fuzzy Probability Approximation*

In order to compute the survival function of a system with components whose random failure times are based on distribution functions with probabilistic distribution parameters modeled by fuzzy numbers, a procedure is needed to handle these in probabilistic models. Beer et al. (2013) provides such a procedure, that is based on a repeated p-box analysis and shown in Fig. 2. Each x_α denotes a α-level set of the fuzzy number $\tilde{x}$, representing an interval parameter of a probability distribution and therefore defining a p-box. This leads to an interval for the failure probability P_f, associated with the same α-level. Repeating this p-box analysis with different α-levels, leads to the fuzzy failure probability $\tilde{P}_f$. Note that this allows an easy identification of acceptable levels of uncertainty for decision-makers in the underlying failure probabilities, e.g. in the design of new systems. For more detailed information, see Beer et al. (2013).

4. Multi-Stage High-Speed Axial Compressor

Axial compressors are complex, multi-component machines that are employed in major sectors of society, e.g., in the industrial sector, as a key component of gas turbines for electricity production or as part of aircraft engines in public transport or the military sector. Therefore, in both, design and maintenance, it is critical to consider as many system performance influencing, certain and uncertain, informations as possible to maximize the compressor's reliability efficiently. In order to illustrate this, the proposed method is applied to a functional model of an axial compressor.

4.1. *Model*

In Miro et al. (2019) a functional model of an axial compressor is developed as the foundation for a reliability analysis. This model has been created to represent the reliability characteristic and functionality of the four-stage high-speed axial compressor of the Institute for Turbomachinery and Fluid Dynamics at Leibniz Universität Hannover. Detailed information about this axial compressor is provided in Braun and Seume (2006); Hellmich and Seume (2008); Siemann et al. (2016).

The functional model captures the dependence of the overall compressor performance, namely the total-to-total pressure ratio and the total-to-total isentropic efficiency, on the surface roughness of the individual blades, arranged in rotor and stator rows. It is based on the results of a sensitivity analysis of the aerodynamic model of the compressor. A network representation of the functional model is shown in Fig. 3. Each component represents either a stator (S1 - S4) or rotor row (R1 - R4).

The rows are classified into four component types. This classification as well as the components' arrangement is chosen based on the resulting effect of their blade roughness on the two

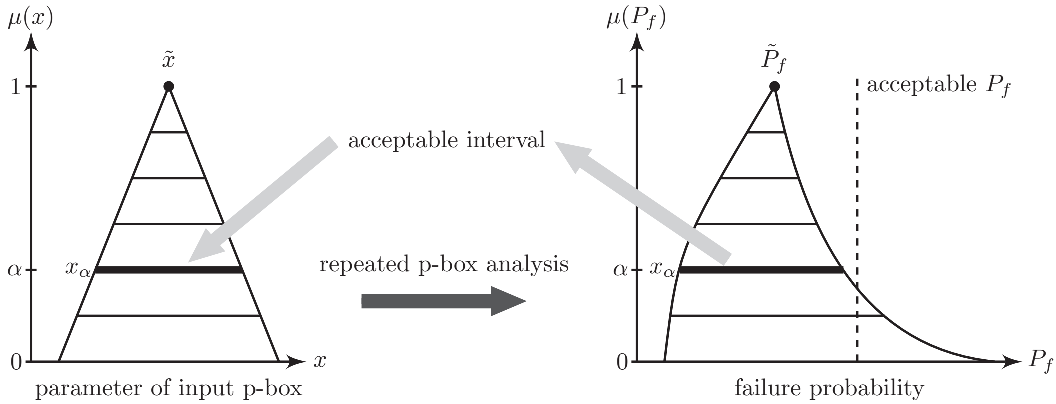

Fig. 2. Nested p-box analysis to determine a fuzzy failure probability; adapted from Ref. Beer et al. (2013)

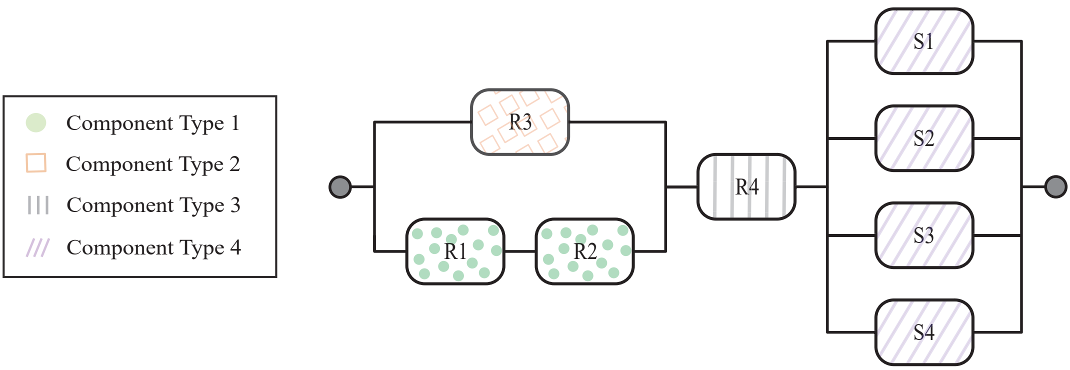

Fig. 3. Functional model of the multi-stage high-speed axial compressor

performance parameters of the axial compressor. More specific, a connection between start and end implies a functional status of the compressor, and an interruption of this connection means exceeding a roughness related performance variation of at least 25%, corresponding to a non-functional status. More detailed information on the functional model and its formulation can be obtained from Miro et al. (2019).

4.2. *Reliability Analysis*

For the time-dependent reliability analysis, each row, i.e. each component of the functional model, is characterized by a failure probability depending on its component type. Normally, the underlying distribution functions have to be derived from existing operational data. However, in order to prove the usability of the proposed method and the capability of dealing with epistemic uncertainties, exponential functions with uncertain parameters are assumed for all components. These parameters are modeled by triangular fuzzy numbers $\lambda_i = (a/b/c)$, with $a < b < c$, $[a, c]$ denoting the base of λ_i and b denoting its vertex, i.e. $\lambda_i(b) = 1$. Depending on the respective component type, the following triangular fuzzy parameters are assumed: $\lambda_1 = (0.2/0.6/1)$ for type 1; $\lambda_2 = (0.3/0.55/0.8)$ for type 2; $\lambda_3 = (0.5/0.6/0.7)$ for type 3; $\lambda_4 = (0.6/0.7/0.8)$ for type 4.

After determining the compressor's survival signature, in a first step, the uncertain parameters are taken into account by approximating them with just a single p-box, being the base of each triangle fuzzy parameter, corresponding to an α-level of $\alpha = 0$. The uncertain parameters result as: $\lambda_1 \in [0.2, 1]$ for type 1; $\lambda_2 \in [0.3, 0.8]$ for type 2; $\lambda_3 \in [0.5, 0.7]$ for type 3; $\lambda_4 \in [0.6, 0.8]$ for type 4. Based on the functional compressor representation showed in Fig. 3, the lower and upper bounds of the compressor's survival function result as displayed in Fig. 4: 1. via LEMCS algorithm with $\lambda_1^* = 0.2$, $\lambda_2^* = 0.3$, $\lambda_3^* = 0.5$, $\lambda_4^* = 0.6$ as the best fits for λ_i^*; 2. via GEMCS algorithm with $p(\boldsymbol{\lambda})$ assumed to be uniform; 3. analytically.

Clearly, both, the LEMCS and GEMCS algorithm, approximate the analytically calculated upper and lower bounds of the survival function accurately. However, when considering the relative errors: $\overline{\delta}_{LEMCS} = 0.036\%$, $\underline{\delta}_{LEMCS} = 0.12\%$ and $\overline{\delta}_{GEMCS} = 1.2\%$, $\underline{\delta}_{GEMCS} = 0.1\%$, with $\overline{\delta}$ relating to the upper and $\underline{\delta}$, respectively, relating to the lower bound of the survival function, it is noticeable that, in comparison to the LEMCS with λ_i^* being fixed on the lower bound of the uncertain parameters, the GEMCS algorithm slightly underestimates the upper bound of the survival function. This is caused by the fact that the samples $\Phi_{si}^{(k)}$ generated by GEMCS have a heavier probability mass on the left than those generated by LEMCS, due to the variation of λ_i in the GEMCS sampling process. Furthermore, it has to be noted that in general the GEMCS algorithm performs more stable than the LEMCS algorithm and the approximation quality of the LEMCS algorithm highly depends on the choice, respectively, on the knowledge of the preselected distribution parameters λ_i^*.

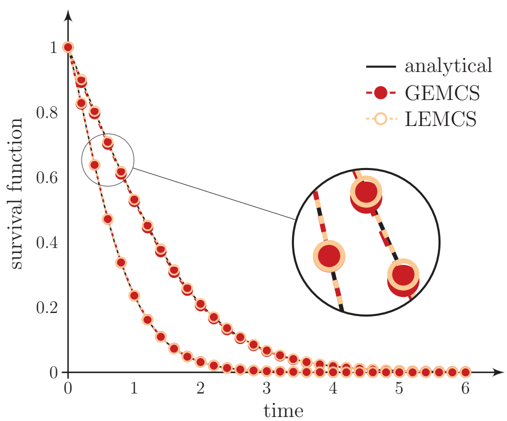

Fig. 4. Survival function bounds of a functional compressor model via LEMCS algorithm, GEMCS algorithm and analytically

In a second step, with the nested p-box analysis to determine fuzzy failure probabilities, described in Section 3.3, further bounds of the survival function for different uncertainty levels can now be determined, based on various α-levels. With regard to the survival signature, these only represent a change in the probability structure. Due to the separation between topological and probability structure, the survival signature does not have to be recalculated for each new α-level, but only the probability structure has to be adapted. This results in a substantial reduction of the computational effort.

The results of the LEMCS algorithm for different α-levels are shown in Fig. 5. Note that for each α-level just one single simulation, according to Sec. 3.1, had to be performed. The results enable decision-makers to estimate in design and maintenance processes of complex capital goods the level of uncertainty that is bearable and still ensures an acceptable reliability.

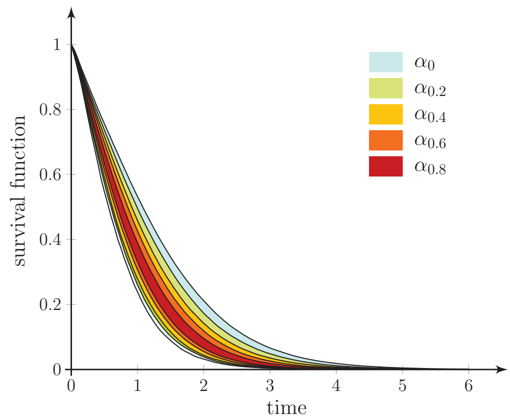

Fig. 5. Survival function bounds of a functional compressor model via LEMCS algorithm with fuzzy probability approximation

5. Conclusion and Outlook

The present paper addresses the development of a methology supporting decision-making processes in the context of system reliability, taking into account epistemic uncertainties. The methology allows to efficiently estimate the system reliability in design and maintenance processes, considering the uncertainty, underlying in the system components' behavior. Thereby, decision-makers are enabled to identify a bearable level of uncertainty that still ensures an acceptable system reliability.

The proposed method integrates the concept of survival signature into two adapted extended MCS methods. Considering imprecision into the probabilistic structure by means of fuzzy probabilities and utilizing a nested p-box analysis for approximating this fuzzyness, allows for the ability of critical uncertainty identification. The application of the concept of survival signature dramatically reduces the computational effort for these nested computations. Another key feature of the novel method is the significantly improved efficiency in the consideration of the system reliability inherented imprecision, accomplished by the main benefit of the adapted NISS methods - the necessity of only a single stochastic simulation per considered uncertainty level.

The novel approach is applied to the functional model of an axial compressor developed in Miro et al. (2019). A comparison of the analytical and numerical results proves the method's applicability. However, in general the LEMCS exhibits more local accuracy, while the GEMCS possesses a higher stability.

Further research should address the applicability to more complex systems. Especially the computation of the survival signature is for larger systems demanding or even unfeasible, thus, improved methods for determining the survival signature or approximations are required. Furthermore, the future work of the authors will address an improved rare failure event estimation and further performance improvements as, e.g, utilization of a HDMR.

Acknowledgement

Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – SFB 871/3 – 119193472

References

Aughenbaugh, J. M. and C. J. J. Paredis (2005, 12). The Value of Using Imprecise Probabilities in Engineering Design. *Journal of Mechanical Design 128*(4), 969–979.

Beer, M., S. Ferson, and V. Kreinovich (2013). Imprecise probabilities in engineering analyses. *Mechanical systems and signal processing 37*(1-2), 4–29.

Braun, M. and J. Seume (2006). Forward Sweep in a Four-Stage High-Speed Axial Compressor. *ASME Turbo Expo 2006: Power for Land, Sea, and Air Volume 6: Turbomachinery, Parts A and B*, 141 – 152.

Buckley, J. J. (2005). *Fuzzy probabilities: new approach and applications*, Volume 115. Springer Science & Business Media.

Coolen, F. P. and T. Coolen-Maturi (2013). Generalizing the signature to systems with multiple types of components. In *Complex systems and dependability*, pp. 115–130. Springer.

Coolen, F. P. and T. Coolen-Maturi (2016). The structure function for system reliability as predictive (imprecise) probability. *Reliability Engineering & System Safety 154*, 180–187.

Der Kiureghian, A. and O. Ditlevsen (2009). Aleatory or epistemic? does it matter? *Structural safety 31*(2), 105–112.

Feng, G., E. Patelli, M. Beer, and F. P. Coolen (2016). Imprecise system reliability and component importance based on survival signature. *Reliability Engineering & System Safety 150*, 116–125.

He, Y., M. Mirzargar, and R. M. Kirby (2015). Mixed aleatory and epistemic uncertainty quantification using fuzzy set theory. *International Journal of Approximate Reasoning 66*, 1–15.

Hellmich, B. and J. Seume (2008). Causes of Acoustic Resonance in a High-Speed Axial Compressor. *ASME. J. Turbomach. 130*(3), 031003–031003–9.

Hofer, E., M. Kloos, B. Krzykacz-Hausmann, J. Peschke, and M. Woltereck (2002). An approximate epistemic uncertainty analysis approach in the presence of epistemic and aleatory uncertainties. *Reliability Engineering & System Safety 77*(3), 229–238.

Miro, S., T. Willeke, M. Broggi, J. Seume, and M. Beer (2019). Reliability Analysis of an Axial Compressor Based on One-Dimensional Flow Modeling and Survival Signature. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering 5*, 031003–1 – 031003–9.

Möller, B., W. Graf, M. Beer, and J. Sickert (2001). Fuzzy probabilistic method and its application for the safety assessment of structures. In *Proceedings of the European Conference on Computational Mechanics, Cracow, Poland*.

Patelli, E., G. Feng, F. P. Coolen, and T. Coolen-Maturi (2017). Simulation methods for system reliability using the survival signature. *Reliability Engineering & System Safety 167*, 327–337.

Siemann, J., I. Krenz, and J. Seume (2016). Experimental Investigation of Aspiration in a Multi-Stage High-Speed Axial-Compressor. *ASME. Turbo Expo: Power for Land, Sea, and Air Volume 2A: Turbomachinery*.

Wei, P., J. Song, S. Bi, M. Broggi, M. Beer, Z. Lu, and Z. Yue (2019a). Non-intrusive stochastic analysis with parameterized imprecise probability models: I. performance estimation. *Mechanical Systems and Signal Processing 124*, 349–368.

Wei, P., J. Song, S. Bi, M. Broggi, M. Beer, Z. Lu, and Z. Yue (2019b). Non-intrusive stochastic analysis with parameterized imprecise probability models: Ii. reliability and rare events analysis. *Mechanical Systems and Signal Processing 126*, 227–247.